Normal vector: $r_u \times r_v = \langle a,\ b,\ c\rangle$

Plane equation: $a(x - x_0) + b(y - y_0) + c(z - z_0)$

Coordinate Systems

Polar: $x = rcos(\theta)$ $y = rsin(\theta)$ $dA = r\,dr\,d\theta$ $x^2 + y^2 = r^2$ $\theta = tan^{-1}\left(\frac{y}{x}\right)$	*Cylindrical:* Polar, except $dA = r\,dz\,dr\,d\theta$	*Spherical:* $x = \rho\,sin(\phi)\,cos(\theta)$ $y = \rho\,sin(\phi)\,sin(\theta)$ $z = \rho\,cos(\phi)$ $dA = \rho^2\,sin(\phi)\,d\rho\,d\phi\,d\theta$ $x^2 + y^2 + z^2 = \rho^2$

Line and Surface Integrals

$$\int_C \overline{F} \cdot d\overline{r} = \int_C \overline{F} \cdot \overline{T}\,ds = \int_a^b F(\overline{r}(t)) \cdot \overline{r}'(t)\,dt$$

$$\int\int_S \overline{F} \cdot d\overline{S} = \int\int_C \overline{F} \cdot \overline{n}\,dS = \int\int_D \overline{F}(\overline{r}(x,y)) \cdot (r_x \times r_y)\,dA$$

$$may\ be \int\int_D \overline{F}(\overline{r}(\phi,\theta)) \cdot (r_\phi \times r_\theta)\,dA$$

$$\int\int_D f(x,y,z)\,dS = \int\int_D f(r(u,v)) \left|r_u \times r_v\right| dA$$

Parameterizations

"For lines when given points P and Q" $\overline{r}(t) = (1 - t)P + tQ$	*"When using polar coordinates"* $\overline{r}(t) = \langle rcos(t),\ rsin(t),\ t\rangle$	*"Easy parameterization"* $\overline{r}(t) = \langle x,\ y,\ f(x,y)\rangle$ $\overline{n} = \langle -f_x,\ -f_y,\ 1\rangle$

Three Theorems

Stokes' $\int_C \overline{F} \cdot d\overline{r} =$ $\int\int_S curl(F) \cdot d\overline{S}$	*Divergence* $\int\int_S \overline{F} \cdot d\overline{S} =$ $\int\int\int_B div(F)\,dV$	*Green's* $\int_C P\,dx + Q\,dy =$ $\int\int_D \left(\frac{\delta Q}{\delta x} - \frac{\delta P}{\delta y}\right)dA$

When

$r(\phi, \theta) = \langle sin(\phi)\, cos(\theta),\ sin(\phi)\, sin(\theta),\ cos(\phi) \rangle$

$r_{\phi} \times r_{\theta} = \langle sin^{2}(\phi)\, cos(\theta),\ sin^{2}(\phi)\, sin(\theta),\ sin(\phi)\, cos(\phi) \rangle$

$\left| r_{\phi} \times r_{\theta} \right| = sin(\phi)$

Half-Angle Identities

$sin^{2}(x) = \frac{1 - cos(2x)}{2}$

$cos^{2}(x) = \frac{1 + cos(2x)}{2}$

Fundamental Theorem of Line Integrals

Given that $f(x, y)$ is a potential equation of a conservative field $\overline{F}$

$\int_{C} \overline{F} \cdot d\overline{r} = f(Q) - f(P)$

Area With No Flux

$A(S) = \int\int_{D} \sqrt{1 + \left(\frac{\delta z}{\delta x}\right)^{2} + \left(\frac{\delta z}{\delta y}\right)^{2}}\, dA$

2nd Derivative Test

$D = f_{xx} f_{yy} - \left(f_{xy}\right)^{2}$

If $D < 0$, then the point is a saddle

If $D = 0$, then the test is inconclusive

If $D > 0$ and $f_{xx} > 0$, then it's a minimum value

If $D > 0$ and $f_{xx} < 0$, then it's a maximum value