Logical Equivalences

Involving Conditional Statements	Involving Biconditional Statements
$p \to q \equiv \neg p \lor q$ $p \to q \equiv \neg q \to \neg p$ $p \lor q \equiv \neg p \to q$ $p \land q \equiv \neg(p \to \neg q)$ $\neg(p \to q) \equiv p \land \neg q$ $(p \to q) \land (p \to r) \equiv p \to (q \land r)$ $(p \to r) \land (q \to r) \equiv (p \lor q) \to r$ $(p \to q) \lor (p \to r) \equiv p \to (q \lor r)$ $(p \to r) \lor (q \to r) \equiv (p \land q) \to r$	$p \leftrightarrow q \equiv (p \to q) \land (q \to p)$ $p \leftrightarrow q \equiv \neg p \leftrightarrow \neg q$ $p \leftrightarrow q \equiv (p \land q) \lor (\neg p \land \neg q)$ $\neg(p \leftrightarrow q) \equiv p \leftrightarrow \neg q$

Useful Tidbits
$A \cup B$ union consists of all elements between the two sets $A \cap B$ intersection consists of shared elements $A - B$ difference consist of the elements of A not in B Power Sets P consist of all combos including $\emptyset$ and itself $\forall x P(x)$ — $P(x)$ is true for every x. $\exists x P(x)$ — There is an x for which $P(x)$ is true. If $a \to b$ Converse: b then a Contrapositive: $\neg b$ then $\neg a$ Inverse: $\neg a$ then $\neg b$

Rules of Inference

Rule of Inference	Tautology	Name
p $p \to q$ $\therefore q$	$(p \land (p \to q)) \to q$	Modus Ponens If Alex enters the failsafe codes into his time machine, he will be sent back to the present. He has entered the failsafe codes, so he is being sent back to the present.
$\neg q$ $p \to q$ $\therefore \neg p$	$(\neg q \land (p \to q)) \to \neg p$	Modus Tollens If Medley is a chicken, then he is a bird; if Medley is not a bird, then Medley is not a chicken.
$p \to q$ $q \to r$ $\therefore p \to r$	$((p \to q) \land (q \to r)) \to (p \to r)$	Hypothetical Syllogism If I do not wear a hazmat suit, I cannot go into the primary reactor chamber. If I cannot go into the primary reactor chamber, I cannot fix the reactor and there will be a meltdown. Therefore, if I do not wear a hazmat suit, then I cannot fix the reactor and there will be a meltdown.
$p \lor q$ $\neg p$ $\therefore q$	$((p \lor q) \land \neg p) \to q$	Disjunctive Syllogism I will choose to visit Ancient Greece or Ancient Rome. I will not choose Ancient Rome, therefore I will choose Ancient Greece.
p $\therefore p \lor q$	$p \to (p \lor q)$	Addition I will eat a delicious sandwich. I will eat a delicious sandwich or I will drink a refreshing glass of water.
$p \land q$ $\therefore p$	$(p \land q) \to p$	Simplification I will learn morse code and nautical flag signals, therefore I will learn morse code.
p q $\therefore p \land q$	$((p) \land (q)) \to (p \land q)$	Conjunction I will escape this time rift. I will fix my time machine. Therefore, I will escape this time rift and fix my time machine.
$p \lor q$ $\neg p \lor r$ $\therefore q \lor r$	$((p \lor q) \land (\neg p \lor r)) \to (q \lor r)$	Resolution I will take a stroll on the beach or I will take a nap in the sun. I will not take a stroll on the beach or my cat and I will go surfing. Therefore, I will take a nap in the sun or my cat and I will go surfing.

Rules of Inference for Quantified Statements

Rule of Inference	Name
$\forall x P(x)$ $\therefore P(c)$	Universal Instantiation Our domain consists of all time travelers and Bill and Ted are time travelers. All time travelers have met Socrates. Therefore, Bill and Ted have met Socrates.
$P(c)$ for an arbitrary c $\therefore \forall x P(x)$	Universal Generalization If you visit an aviary and notice that all the parrots are colorful, then you may assume that all parrots are colorful.
$\exists x P(x)$ $\therefore P(c)$ for some element c	Existential Instantiation There is someone who has eaten a delicious dinner in a dungeon. Let's call her Marcille and say that Marcille has eaten a delicious dinner in a dungeon.
$P(c)$ for some element c $\therefore \exists x P(x)$	Existential Generalization This dish of bread, rice, beans, greens, and breaded meat with a side of home-baked cake is appetizing. Therefore, an appetizing meal exists.

Fallacies

Denying the Hypothesis	$p \rightarrow q$ and $\neg p$, $\therefore \neg q$ If you look for tripwires, then you won't fail the mission. You failed the mission, therefore you didn't look for tripwires. This is a fallacy because you could have failed the mission for a variety of other reasons, i.e. tripping the sensors, fumbling the EMP device, or not wiping your boots before propelling down the building.
Affirming the Conclusion	$p \rightarrow q$ and q, $\therefore p$ If Alex is cold, then he will turn on the AC. If Alex turns on the AC, then he is cold. This is a fallacy because sometimes Alex isn't cold every time he turns on the AC. Someone else could have asked him to or there could have been the smell of smoke in the room.
Begging the Question & Circular Reasoning	$p \rightarrow q \therefore q \rightarrow p$ This restaurant is the best in town because their menu says so. The menu itself is made by the restaurant.

Proofs

Direct	If a shape is a square, then it has four sides. If the shape has four sides, then it is a quadrilateral. Thus, a square is a quadrilateral.
By Contraposition	If it is raining, then I wear my coat — if I don't wear my coat, then it isn't raining.
Vacuous and Trivial	Let $x \in R$. If $x < 0$, then $x^2 + 1 > 0$. It's trivial because x^2 will always be positive.
Contradiction	Assume the opposite of what you want to prove.
Equivalence	$p = q$