Useful Equations

Independent Events $\quad P(A|B) = P(A)$

Pick (Permutations) and Choose (Combinations)

$${}_nP_k = \frac{n!}{(n-k)!} \qquad {}_nC_k = \binom{n}{k} = \frac{n!}{k!(n-k)!}$$

Bayes' Theorem

$$P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{P(B|A) * P(A)}{P(B)} = \frac{P(B|A) * P(A)}{P(B|A) * P(A) + P(B|A^c) * P(A^c)}$$

Inclusion-Exclusion

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

Binomial Distribution

$$X \sim Bin(n,k) = \binom{n}{k} p^k * (1-p)^{n-k} \qquad E(X) = np \qquad V(E) = np(1-p)$$

CDF Integration Rule of $f_X(x)$ $\quad$ Expected Value of $f_X(x)$

$$\int_{-\infty}^{\infty} f_X(x)\,dx = 1 \qquad\qquad \int_{-\infty}^{\infty} x f_X(x)\,dx$$

Marginal Density

$$f_X(x) = \int_{-\infty}^{\infty} f_{X,Y}(x,y)\,dy \qquad f_Y(y) = \int_{-\infty}^{\infty} f_{X,Y}(x,y)\,dx$$

X and Y aren't independent if $f_{X,Y}(x,y) \neq f_X(x) f_Y(y)$ for at least one (x,y)

$$Var(X) = E(X^2) - E(X)^2, \quad Var(X+Y) = Var(X) + Var(Y) + 2Cov(X,Y), \quad Cov(X,Y) = E(XY) - E(X)E(Y)$$

$$E(XY) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} xy\ f_{X,Y}(x,y)\ dA\ (x,y)$$

Let X be a random variable. The Moment Generating Function of X is

$$M_X(t) = \sum_{k=0}^{\infty} e^{tk}\ p(k) \text{ when } X \text{ is discrete and } M_X(t) = \int_{-\infty}^{\infty} e^{tk}\ f_X(x)\ dx \text{ when } X \text{ is continuous}$$

If X and Y are independent, then $M_{X+Y}(t) = M_X(t) \cdot M_Y(t)$

Derivative power rule of a function: $\frac{d}{dx} f(x)^n = n f(x)^{n-1} f'(x)$

$$P(Y \leq y | X = x) = \frac{\int_{-\infty}^{y} f_{X,Y}(x,u)\,du}{f_X(x)}$$

$$M_x''(0) = E(X^2) \qquad P(Z \geq X) = 1 - F_Z(X) \qquad F(Z < X) = F_Z(X) \qquad \Gamma(1,\lambda) = Exp(\lambda) \qquad \Gamma(1) = 1 \qquad \Gamma(\frac{1}{2}) = \sqrt{\pi}$$