Problem 1

A computer program can make calls to two subroutines, A and B. In a randomly chosen run, let X be the number of calls to subroutine A and Y be the number of calls to subroutine B. The joint probability mass function of X and Y is given by the following table.

(a) Find the marginal probability mass functions of X and Y.

X/Y	0	1	2	
1	0.1	0.05	0.1	0.25
2	0.1	0.15	0.05	0.30
3	0.15	0.2	0.1	0.45
	0.35	0.40	0.25	

(b) Find $E(X)$ and $E(Y)$.

$E(X) = 1 \cdot 0.25 + 2 \cdot 0.30 + 3 \cdot 0.45 = 2.2$
$E(Y) = 0 \cdot 0.35 + 1 \cdot 0.40 + 2 \cdot 0.25 = 0.9$

(c) Find $Cov(X, Y)$.

$Cov(X, Y) = E(XY) - E(X)E(Y) = 1.95 - 2.2 \cdot 0.9 = -0.03$
$E(XY) = 1 \cdot 1 \cdot 0.05 + 1 \cdot 2 \cdot 0.1 + 2 \cdot 1 \cdot 0.15 + 2 \cdot 2 \cdot 0.05 + 3 \cdot 1 \cdot 0.2 + 3 \cdot 2 \cdot 0.1 = 1.95$

(d) Are X and Y independent? Explain.

X and Y are not independent as $0.25 \cdot 0.35 \neq 0.1$

(e) Find $P(X > 1.5 \text{ and } Y \leq 1)$

$P(X > 1.5 \text{ and } Y \leq 1) = 0.1 + 0.15 + 0.15 + 0.2 = 0.6$

Problem 2

Let X and Y be continuous random variables with joint probability density function

$$f_{x,y}(x, y) = \begin{cases} \frac{3}{2}y^2, & 0 \leq x \leq 2 \text{ and } 0 \leq y \leq 1 \\ 0, & \text{otherwise} \end{cases}$$

(a) Find the marginal densities of X and Y (make sure to give the regions where they are valid).

$$f_x(x) = \int_{-\infty}^{\infty} f_{x,y}(x, y)\, dy = 0 + \int_0^1 \frac{3}{2}y^2\, dy = \frac{3}{2}\int_0^1 y^2\, dy = \frac{3}{2}\left(\frac{y^3}{3}\right)\Big|_0^1 = \frac{1}{2}$$

$$f_y(y) = \int_{-\infty}^{\infty} f_{x,y}(x, y)\, dx = 0 + \int_0^2 \frac{3}{2}y^2\, dx = \frac{3}{2}y^2 \int_0^2 1\, dx = 3y^2$$

$$f_x(x) = \begin{cases} \frac{1}{2}, & 0 \leq x \leq 2 \\ 0, & \text{otherwise} \end{cases} \qquad f_y(y) = \begin{cases} 3y^2, & 0 \leq y \leq 1 \\ 0, & \text{otherwise} \end{cases}$$

(b) Are X and Y independent? Explain your answer.

X and Y are independent as $\frac{2}{3}y^2 = \frac{1}{2} \cdot 3y^2$. For future reference, this means that $Cov(X, Y) = 0$

(c) Find the expected value and variance of Y.

$$E(Y) = \int_{-\infty}^{\infty} y f_y(y)\, dy = \frac{3}{4} \qquad E(Y^2) = \int_{-\infty}^{\infty} y^2 f_y(y)\, dy = \frac{3}{5}$$

$$V(Y) = E(Y^2) - E(Y)^2 = \frac{3}{80}$$

(d) Find the variance of $X+Y, Var(X+Y)$.

$Var(X+Y) = Var(X) + Var(Y) + 2Cov(X,Y)$, but since $Cov(X,Y) = 0$ by their independence,

$Var(X+Y) = Var(X) + Var(Y)$

$Var(X) = E(X^2) - E(X)^2 = \frac{4}{3} - 1 = \frac{1}{3}$ $Var(X+Y) = \frac{3}{80} + \frac{1}{3} = \frac{89}{240}$

(e) Calculate $P\Big(X < \frac{3}{4}\Big|Y = \frac{1}{2}\Big)$.

$$P(Y \le y|X = x) = \frac{\int_{-\infty}^{y} f_{X,Y}(x,u)\, du}{f_X(x)}; \quad P\Big(X \le \frac{3}{4}|X = x\Big) = \frac{\int_0^{3/4} f_{X,Y}(u, \frac{1}{2})\, du}{f_Y(\frac{1}{2})}$$

(f) Compute $E\Big[X|Y = \frac{1}{2}\Big]$.

$E\Big(X|Y = \frac{1}{3}\Big) = E(X)$ as X and Y are independent, therefore $E\Big(X|Y = \frac{1}{3}\Big) = 1$.

(g) Compute the joint CDF of X and Y, $F_{X,Y}(X,Y)$ Evaluate all the cases:

I)

When $x \le 0, \quad y \le 0, \quad F_{X,Y}(x,y) = 0$

II)

When $0 \le x \le 2, y > 1$, $F_{X,Y}(x,y) = \int_0^x \int_0^1 \frac{3}{2}u^2\, du\, dv = \frac{x}{2}$

III)

When $x > 2, 0 \le y \le 1$, $F_{X,Y}(x,y) = \int_0^2 \int_0^y \frac{3}{2}u^2\, du\, dv = y^3$

IV)

When $0 \le x \le 2, 0 \le y \le 1$, $F_{X,Y}(x,y) = \int_0^x \int_0^y \frac{3}{2}u^2\, du\, dv = \frac{xy^3}{2}$

V)

When $x > 0, \quad y > 0, \quad F_{X,Y}(x,y) = 1$

Therefore,

$$F_{X,Y}(X,Y) = \begin{cases} 0, & 0 \le x \le 0, y \le 0 \\ \frac{x}{2}, & x \le 2, y > 1 \\ y^3, & x > 2, 0 \le y \le 1 \\ \frac{xy^3}{2}, & 0 \le x \le 2, \quad 0 \le y \le 1 \\ 1, & x > 0, y > 0 \end{cases}$$

Problem 3

Let Y be a uniform random variable on $[0,1]$. Let $W = Y^2$. Find the PDF of W.

$Y \sim U([0,1])$

$$f_y(y) = \begin{cases} \dfrac{1}{length[0,1]}, & 0 \le y \le 1 \\ 0, & \text{otherwise} \end{cases} \longrightarrow f_y(y) = \begin{cases} 1, & 0 \le y \le 1 \\ 0, & \text{otherwise} \end{cases}$$

$W = Y^2 \rightarrow$ pdf of W

Y given using pdf $\rightarrow$ cdf of Y $\rightarrow$ cdf of $W = Y^2$, we want the pdf

$$F_y(y) = P(Y \le y) = \int_{-\infty}^{\infty} f_y(t)\, dt = \begin{cases} \int_{-\infty}^{y} 0\, dt = 0, & y < 0 \\ \int_{-\infty}^{y} f_y(t)\, dt = \int_0^y 1\, dt = y, & 0 \le y \le 1 \\ \int_{-\infty}^{y} f_y(t)\, dt = \int_0^1 1\, dt = 1, & y > 1 \end{cases} = \begin{cases} 0, & y < 0 \\ y, & 0 \le y \le 1 \\ 1, & y > 1 \end{cases}$$

$F_w(w) = P(W \le w) = P(Y^2 \le w) = 0$ if $w < 0$

$F_w(w) = P(Y^2 \le w) = P(Y \le \sqrt{w}) = F_y(\sqrt{w})$ if $w \ge 0$

$P(Y^2 \le w) = P(-\sqrt{w} \le Y \le \sqrt{w}$

If $W \in [0,1] : F_y(\sqrt{w}) = \sqrt{w}$

If $W > 1 : F_y(\sqrt{w}) = 1$

$$F_w(w) = \begin{cases} 0, & w < 0 \\ \sqrt{w}, & 0 \le w \le 1 \\ 1, & w > 1 \end{cases} \longrightarrow F_w'(w) = \begin{cases} 0, & w < 0 \\ \dfrac{1}{2\sqrt{w}}, & 0 \le w \le 1 \\ 0, & w > 1 \end{cases}$$

Problem 4

Let $X_1 \sim Poisson(\lambda_1)$ and $X_2 \sim Poisson(\lambda_2)$, and assume that X_1 and X_2 are independent. Recall that the PDF of a Poisson random variable with parameter λ is $p_X(k) = P(X = k) = \dfrac{e^{-\lambda}\lambda^k}{k!}$ for $k = 0, 1, 2, ...$

(a) Derive the moment generating function of X_1.

$p_x(k) = P(X = k) = \dfrac{e^{-\lambda}\lambda^k}{k!}$ for $k = 0, 1, 2, ...$

Note that $e^r = \sum_{k=0}^{\infty} \dfrac{r^k}{k!}$

For discrete functions, $M_t(t) = \sum e^{tk} p(x)$

$$\sum_{k=0}^{\infty} \frac{e^{tk} e^{-\lambda}\lambda^k}{k!} = e^{-\lambda} \sum_{k=0}^{\infty} \frac{(e^t\lambda)^k}{k!}$$

Apply the principle stated above (McLauren Series).

$e^{-\lambda} e^{\lambda e^t} = e^{\lambda(e^t - 1)}$

(b) Find the moment generating function of the sum $X_1 + X_2$. What is the distribution of $X_1 + X_2$?

If X and Y are independent, $M_{X+Y}(t) = M_X(t) M_Y(t)$

$e^{\lambda_1(e^t-1)} e^{\lambda_2(e^t-1)} = e^{\lambda_1(e^t-1) + \lambda_2(e^t-1)} = e^{(e^t-1)(\lambda_1+\lambda_2)}$

Problem 5

Let X and Y be random variables with finite variances.

(a) What is the definition of the covariance of X and Y, $Cov(X, Y)$?

The $Cov(X, Y)$ measures how similarly they trend.

(b) True or False? If $Cov(X, Y) = 0$, then X and Y are independent. Explain your answer.

If $Cov(X, Y) = 0$, then X and Y aren't necessarily independent. To be independent, the variables must follow the rule $f_{x,y}(x, y) = f_x(x)f_y(y)$ for all (x, y).

Problem 6

Let X be a binomial random variable with parameters n and p. Find the 2^{nd} (about the origin) of X using the moment generating function $M_x(t) = (1 - p + pe^t)^n$.

$M'_x(t) = n(1 - p + pe^t)^{n-1}(pe^t)$

$(f(x)g(x))' = f(x)g'(x) + f'(x)g(x)$

$(pe^t)((n-1)(1-p+pe^t))^{n-2}(pe^t)) + (pe^t)(1-p+pe^t)^{n-1}$, plug in $t = 0$
$(p)((n-1)) + p = p(p(n-1)+1) = p^2(n-1)p$, multiply by n

$np^2(n-1) + np$

Another way of solving this problem is via this rule:

$V(X) = E(X^2) - E^2(X), \quad E(X^2) = V(X) + E^2(X)$

By using the sheet with all of the information on the different distributions, plug in the corresponding values to get $np(1-p) + (np)^2$.

Problem 7

Let Y be an exponential random variable with parameter (rate) λ. Find the 2nd moment (about the origin) of Y using the moment generating function $M_y(t) = \dfrac{\lambda}{\lambda - t}$.

$Y \sim Exp(\lambda), M_y(t) = \dfrac{\lambda}{\lambda - t}, \lambda(\lambda - t)^{-1}$

2^{nd} moment of $Y = \in (Y^2) = M''_y(0)$

$M'_y(t) = \lambda(-1)(\lambda - t)^{-1-1}(-1) = \dfrac{\lambda}{(\lambda - t)^2} = \lambda(\lambda - t)^{-2}$

Derivative power rule: $(f^n(t))' = nf^{n-1}(t)f'(t)$

$M''_y(t) = \lambda(-2)(\lambda - t)^{-2-1}(-1) = 2\lambda(\lambda - t)^{-3} = \dfrac{2\lambda}{(\lambda - t)^3}$

$M''_y(0) = \dfrac{2\lambda}{(\lambda - 0)^3} = \dfrac{2}{\lambda^2}$

Note: "about the origin" indicates the 0 in $E((Y - 0)^2)$

Problem 8

Let $X_1, ..., X_n$ be a random sample of size n from a geometric distribution with success probability p (that is, let X_i be i.i.d. geometric RVs with parameter p for $i = 1, ..., n$). Let

$$X = \sum_{i=1}^{n} X_i$$

What is the distribution of X? What is the expected value of X? Explain your answer.

$X_1, X_2, ..., X_n$ are a sample (of size n). Independent identical distribution (i.i.d.) is implied because of the word "sample".

$S_n = X_1 + X_2 + ... + X_n$, the distribution of S_n is the Negative Binomial. The distribution and expected value come from the table.

$PMF = \binom{x-1}{n-1} p^n (1-p)^{x-n}$ with $E(X) = \dfrac{n}{p}$

Problem 9

A random sample of size $n = 36$ is drawn from the PDF $f_Y(y) = 2y \;\text{ for }\; y \in [0, 1]$. Let

$$\overline{Y} = \frac{1}{36} \sum_{i=1}^{18} Y_i.$$

Use the Central Limit Theorem to approximate the probability of $P(a \leq \overline{Y} \leq b)$.

$$Y \;\text{ with pdf }\; f_y(y) = \begin{cases} 2y, & 0 \leq y \leq 1 \\ 0, & \text{otherwise} \end{cases}$$

i)

$$E(Y) = \int_0^1 y f_y(y) dy = \int_0^1 2y^2 = \frac{2}{3}$$

ii)

$$E(Y^2) = \int_0^1 y^2 f_y(y) dy = \int_0^1 2y^3 = \frac{1}{2}$$

iii) $V(Y) = E(Y^2) - E^2(Y) = \dfrac{1}{2} - \dfrac{4}{9} = \dfrac{1}{18}$

iv) $\sigma(y)$ standard deviation $\dfrac{1}{\sqrt{18}} = \dfrac{\sqrt{2}}{6}$

$Y_1, Y_2, ..., Y_n$ sample $\rightarrow \overline{Y} = \dfrac{Y_1 + Y_2 + ... + Y_n}{n}$

$n = 36$, n is big enough, use the Central Limit Theorem

$\overline{Y} \sim N(n\frac{2}{3}, n\frac{1}{18}) = N(24, 2) = N(24, \sqrt{2}^2)$

$P(a \leq \overline{Y} \leq b) = P(\dfrac{a - 24}{\sqrt{2}} \leq \dfrac{\overline{Y} - 24}{\sqrt{2}} \leq \dfrac{b - 24}{\sqrt{2}})$

Problem 10

The lifetime in days of a certain fuse has exponential distribution with parameter $\lambda = 0.2$.

(a) Find the probability that such a fuse lasts longer than 6 days.

$$F_L(X) = P(L \leq X) = 1 - e^{-\frac{1}{5}x}, P(L < 6) = 1 - F(6) = e^{-\frac{6}{5}} = 0.3011 = 30.11\%$$

(b) What is the mean lifetime of such fuse?

$$E(L) = \frac{1}{0.2} = 5$$

(c) What is the standard deviation of the lifetime of such fuse?

$$\sigma(L) = \frac{1}{\lambda} = \frac{1}{0.2} = 5$$

(d) Ten such fuses are chosen randomly (suppose they are operating independently). Find the probability that at least 3 of them last longer than 6 days.

$X \sim Bin(10, 0.3011)$. $P(X \geq 3) = 1 - [P(X = 0) + P(X = 1) + P(X = 2)] = 62.01\%$

(e) In such a system, when a fuse burns, is replaced by another fuse. What is the probability that using two such fuses the system works after 12 days?

$P(L_1 + L_2 > 12)$. Two exponential distributions added follow a gamma distribution.

$L_1 \sim Exp(0.2), L_2 \sim Exp(0.2) \rightarrow L_1 + L_2 = Gamma(1 + 1, 0.2)$ as $Exp(a) = Gamma(1, a)$

$$Gamma(2, 0.2) \rightarrow f(x) = \frac{0.2^2}{\Gamma(2)} x^{2-1} e^{-0.2x} = 0.02xe^{-\frac{x}{5}}$$

$$P(L_1 + L_2 > 12) = \int_{12}^{\infty} 0.02xe^{-\frac{x}{5}}\, dx \qquad \text{Use integration by parts.}$$

(f) In such a system, when a fuse burns, is replaced by another fuse, and so on. What is the probability that using 64 such fuses the system works after 315 days?

$P(S_{64} \geq 315) = P(\frac{S_{64} - 320}{40} \geq \frac{315 - 320}{40})$ as the Central Limit Theorem states that $\sim N(64 \cdot 5, 64 \cdot 25)$

$= P(Z \geq -0.125) = 1 - F_Z(-0.125) = 0.54973 = 54.97\%$

Problem 11

The thickness of the paper used for a 100-sheet book has the mean 0.08 mm and the standard deviation 0.01 mm. Assume that this book has no covers.

(a) What is the probability that one page selected at random in one such book is thicker than 0.08 mm?

$P(X > 0.08)$ is impossible to compute because no distribution was specified.

(b) What is the probability that a randomly selected book is more than 8.1 mm thick?

$P(S_{100} > 8.1) = 15.87\%$

Since $n = 100$, it is big enough and follows the CLT. The new mean will be $nE(X) = 100 * 0.08 = 8$ and the new variance will be $n\sigma^2(x) = 100 * 0.01^2 = 0.1^2$ or by directly using the standard deviation, $\sqrt{n}\,\sigma(x) = 0.1$

Then $P(S_{100} > 8.1) = P(\frac{S_{100} - 8}{0.1} > \frac{8.1 - 8}{0.1}) = P(Z > 1) = 1 - F_Z(1) = 0.1587 = 15.87\%$

(c) What is the 95^{th} percentile of the book thicknesses?

$a = 95^{\text{th}}$ percentile thickness

$$P(S_{100} \leq a) = 0.95, \quad P\Big(Z \leq \frac{a-8}{0.1}\Big) = 0.95, \quad F_Z\Big(\frac{a-8}{0.1}\Big) = 0.95$$

Convert 0.95 to its standard Z score by using the table or $2^{\text{nd}} \rightarrow$ vars (dist) $\rightarrow$ invNorm with $area =$ the percentile you seek on the TI.

$$\frac{a-8}{0.1} = 1.645, \quad a = 8.1645 \text{ mm}$$

(d) If four books are selected randomly, what is the probability that at most three of them are more than 8.1 mm thick?

$$P(Y \leq 3) = 1 - P(Y = 4) = 1 - \tbinom{4}{4}(0.1587)^4 = 0.9994 = 99.4\%$$

(e) If two books are selected randomly, what is the probability that the difference of their thicknesses is less than 0.05 mm?

$P(|S'_{100} - S''_{100}| < 0.05)$, denote $T = S'_{100} - S''_{100}$, $S'_{100} \sim N(8, 0.1^2), S''_{100} \sim N(8, 0.1^2)$

Refer to this table when adding distributions:

X_1&X_2 Dist.	Resulting Dist.
Normal	Normal
Poisson	Poisson
Binomial	Binomial
Gamma	Gamma
Geometric	Negative Binomial($r = 2$, p)
Exponential	Gamma

$$N(8-8, 0.1^2 + 0.1^2), \; T \sim N(0, 0.02), \mu = 0, \sigma = \sqrt{0.02} = 0.1415$$

$P(|T| < 0.05) = P(-0.05 < t < 0.05) = P(\frac{-0.05-0}{0.14} < \frac{T-0}{0.14} < \frac{0.05-0}{0.14}) = P(-0.357 < Z < 0.357) = F_Z(0.357) - F_Z(-0.357) = 0.2789 = 27.89\%$

Problem 12

The number of cars arriving at a given intersection follows a Poisson distribution with a mean rate of 4 per second.

(a) What is the probability that 4 cars arrive in any given second?

$$Poisson(rT) = e^{-rT}\frac{(rT)^x}{x!}$$

$$P(X = 3) = e^{-4(1)}\frac{(4(1))^3}{8!} = 0.1954 = 19.54\%$$

(b) What is the probability that 8 cars arrive in 3 seconds?

$$P(X = 8) = e^{-4(3)}\frac{(4(3))^8}{8!} = 0.0655 = 6.55\%$$

Problem 13

A teenager is trying to get a driver's license. Let X be the number of tries he needs to pass the road test, and assume that his probability to pass the exam is 0.2 on any given attempt.

(a) Determine the probability mass function of X.

$(0.2)(0.8)^{x-1}$ is a Geometric distribution

(b) On the average, how many attempts is he likely to require before he gets his license?

$$\frac{1}{p} = \frac{1}{0.2} = 5 \text{ attempts}$$

(c) Evaluate $P(\pi < X < \pi^2)$.

$P(\pi < X < \pi^2) = P(3.14 < X < 9.86) \rightarrow P(4 \leq X \leq 9)$ (adjust the range bounds to integers and to $\leq$ conditions to fit the theorem)

$$\sum_{k=4}^{9} 0.2(0.8)^{k-1} = 0.3778 = 37.78\%$$

Problem 14

A door to door salesperson is required to document five in-home visits each day. Suppose that she has a 25% chance of being invited into any given home, with each address representing an independent trial. What is the probability that she requires fewer than 10 houses to achieve her 5^{th} success? On average how many houses she has to visit to achieve her 5^{th} success?

(a) Negative Binomial Distribution $\binom{x-1}{n-1} p^n (1-p)^{x-n}$

$$P(X < 10) = P(X = 5) + P(X = 6) + P(X = 7) + P(X = 8) + P(X = 9) = 0.0489 = 4.89\%$$

(b) On average she has to visit $\dfrac{n}{p} = \dfrac{5}{0.25} = 20$ houses

Problem 15

Let X be the number of packages being mailed by a randomly selected customer at a certain shipping facility. Suppose the distribution of X is as follows:

x	1	2	3
p	0.1	0.4	0.5

Consider a random sample of size n (n customers), with the numbers of packages $X_1, X_2, ..., X_n$, and let $\overline{X} = \dfrac{X_1 + X_2 + ... + X_n}{n}$ be the sample mean number of packages shipped.

Part A: Suppose $n = 2$

(a) Determine the joint probability mass function of (X_1, X_2).

X_1, X_2, are random variables, $P(X_1 = 1, X_2 = 1) = P(X_1 = 1)P(X_2 = 1)...$

X_1/X_2	1	2	3
1	0.01	0.04	0.05
2	0.04	0.16	0.2
3	0.05	0.2	0.25

(b) What is the probability that the two customers will ship the same number of packages?

$$P(X_1 = X_2) = P(X_1 = X_2 = 1) + P(X_1 = X_2 = 2) + P(X_1 = X_2 = 3) = 0.01 + 0.16 + 0.25 = 0.42$$

(c) Let A denote the event that one of the customers will ship two more packages than the other customer. Calculate the probability of this event.

$$P(X_1 = 3, X_2 = 1) + P(X_1 = 1, X_2 = 3) = 0.05 + 0.05 = 0.1$$

(d) Determine the probability distribution of $T_0 = X_1 + X_2$.

Distribution of $X_1 + X_2$

$X_1 + X_2$	2	3	4	5	6
prob	0.01	0.08	0.26	0.4	0.25

(e) What is the probability that the total number of packages shipped by these two customers is at least 4?

$P(X_1 + X_2 = 4) + P(X_1 + X_2 = 5) + P(X_1 + X_2 = 6) = 0.26 + 0.4 + 0.25 = 0.91$

Part B: Suppose $n = 50$

$\overline{X}, \text{CLT} \sim \Big(E(X), \frac{V(X)}{50}\Big)$

(g) Evaluate $P(2.1 < \overline{X} < 2.4)$

$E(X) = 1 \cdot 0.1 + 2 \cdot 0.4 + 3 \cdot 0.5 = 2.4$

$E(X^2) = 1^2 \cdot 0.1 + 2^2 \cdot 0.4 + 3^2 \cdot 0.5 = 6.2$

$Var(X) = E(X^2) - E^2(X) = 0.44, \quad \sigma(x) = \sqrt{0.44} = 0.663$

$$\mu_{\overline{X}} = 2.4, \quad \sigma_{\overline{X}} = \frac{0.663}{\sqrt{50}} = 0.094$$
$$P\Big(\frac{2.1 - 2.4}{0.094} < Z < \frac{2.4 - 2.4}{0.094}\Big) = P(-3.19 < Z < 0)$$
$$= F_Z(0) - F_Z(-3.19) = 0.4993 = 49.93\%$$

Part C: Suppose $n > 30$

(h) Determine n such that $P(X < 2.29) = 0.1587$.

$n =?\ \ n > 30, P(\overline{X} < 2.29) = 0.1587$

$$P\Big(\frac{\overline{X} - \mu}{\frac{\sigma}{\sqrt{n}}} < \frac{2.29 - \mu}{\frac{\sigma}{\sqrt{n}}}\Big) = 0.1587$$
$$P(Z < \frac{2.29 - \mu}{\frac{\sigma}{\sqrt{n}}}) = P(Z < -1)$$
$$2.29 - \mu = \frac{\sigma}{\sqrt{n}}$$
$$2.29 - 2.4 = \frac{0.663}{\sqrt{n}}, \quad \lfloor n \rfloor = 36$$